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Neutrino Masses and a Fourth Generation of Fermions

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Abstract

We study neutrino mass generation in models with four chiral families of leptons and quarks and four right handed neutrinos. Generically, in these models there are three different contributions to the light neutrino masses: the usual see-saw contribution, the tree-level contribution due to mixing of light neutrinos with neutrino of the fourth generation, and the two loop contribution due to the Majorana mass term of the fourth neutrino. We study properties of these contributions and their experimental bounds. The regions of the parameters (mixings of the fourth neutrino, masses of RH neutrino components, etc.) have been identified where various contributions dominate. New possibilities of a realisation of the flavour symmetries in the four family context are explored. In particular, we consider applications of the smallest groups, e.g. SG(20,3), with irreducible representation $\underline{\mathbf{4}}$.

Keywords: neutrino mass generation; fourth generation; flavor symmetry

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1 Introduction

There are various arguments in favour of the existence of a fourth Standard Model (SM) generation of fermions.

- A fourth generation can alleviate the tension between the lower bound on the Higgs mass from LEP II and the fit of the electroweak precision data, which predicts a light Higgs particle [1]. Indeed, the mass splittings between the fourth generation fermions can lead to a negative contribution to the S parameter, which allows for a heavier Higgs. The flavour sector with four families has been thoroughly analysed in [2–5].

- The enlarged CKM matrix contains additional CP phases and naturally leads to more CP violation which can explain the deviations from the predicted SM values in some measurements in B physics [2; 6].

- A fourth generation has been suggested as an explanation of the anomalous like-sign di-muon charge asymmetry [7].

- A fourth generation makes viable electroweak baryogenesis, which is not possible in the SM. The introduction of a further generation leads to additional CP violating phases in the quark mixing matrix (CKM matrix), which are not constrained by experiment yet, *e.g.* see [8] for an analysis of the neutron electric dipole moment, and can lead to a large enough CP violation [9]. In addition, it has been shown, that a strong first order phase transition is possible within the SUSY version of a model with four SM generations (SM4) [10] as well as in a strongly coupled version with dynamical breaking of the electroweak symmetry [11].

- Being similar to top quark condensate models [12], dynamical electroweak symmetry breaking is possible in the context of four generations [13; 14].

- A fourth generation neutrino can contribute to the dark matter density of the Universe if an additional $B - 4L_4$ symmetry is introduced protecting the fourth generation neutrino from decaying and it couples to the three light generations via the new Z' to quarks [15].

- Under the assumption of minimal flavour violation, a fourth generation suppresses proton decay and enforces the R-parity in the context of the MSSM due to the mismatch of numbers of flavours and colours [16].

Significant interest to the fourth generation is also revived due to operation of the Large Hadron Collider (LHC). The LHC can provide a critical test of existence of the fourth generation: either discover or exclude it. (See [17] for a recent review and [18] for an earlier review.) Indeed, the LHC can test the region of fourth generation quark masses, $300 - 800$ GeV, [19], which covers the complete parameter space determined by the partial wave unitarity upper limit of 550 GeV for a quark doublet [20] and the limit obtained in models of a strongly coupled fourth generation. The CMS Collaboration put a lower bound on the mass of fourth generation up-type quark t' of $m_{t'} \gtrsim 450\,\text{GeV}$ [21] and exclude fourth generation down-type quark b' in the mass region $255\,\text{GeV} < m_{b'} < 361\,\text{GeV}$ at 95%

C.L. [22]. Existence of the fourth generation chiral leptons without fourth generation quarks looks rather unnatural and in fact this will require further complication of model to cancel the anomalies.

The parameter space of the fourth generation can also be probed by looking for the Higgs signals [23] (this has also been studied in the MSSM [24]). Currently, the Higgs boson with mass m_H in SM4 with one Higgs doublet is excluded in the region $120\,\mathrm{GeV} < m_H < 600\,\mathrm{GeV}$ at 95% C.L. by CMS [25] and $140\,\mathrm{GeV} < m_H < 185\,\mathrm{GeV}$ by ATLAS [26]. However, these bounds only apply in the minimal SM4 model with one Higgs doublet. They are weakened if (i) the Higgs production via gluon fusion is modified, e.g. by a colour octet [27] or if the light Higgs in a two Higgs doublet model does not couple to the fourth generation [28], (ii) the search channels $h \to WW^*, ZZ^*$ are modified, e.g. in a two Higgs doublet model [14], or (iii) the Higgs decays dominantly invisibly, e.g. into a light scalar, which can provide a dark matter candidate [29] or into fourth generation neutrinos for light Higgs with $m_H < 170\mathrm{GeV}$ [30]. Recently, the complete electroweak two-loop corrections to Higgs production via gluon fusion have been calculated and discussed in the framework of a fourth generation [31]. In the SM with four generations and one Higgs doublet, the Higgs bounds can be translated in a bound on the fourth generation fermion masses via the triviality and stability bounds [32].

Collider signals of the 4th generation have been studied which include signals of fourth generation quarks [19; 23; 33; 34], leptons [35–37], sleptons [38], signals of a strongly coupled generation [39] and a Z' [40]. If the mixing of the fourth generation with the three SM generations is tiny, the particles of the fourth generation become long-lived [41]. The fourth generation quarks might even form long-lived bound states which can be produced at the LHC. The binding energies and sizes of those bound states have been calculated in [42].

Note that SM4 is constrained by the large Yukawa couplings running into Landau poles due to a quick renormalisation group (RG) evolution. The current experimental bounds require a cutoff scale $\Lambda_c \lesssim (10^2 - 10^3)\,\mathrm{TeV}$, unless there is a fixed point [43]. Similar results have been obtained in the SUSY context in [17; 44].

Properties of the fourth generation particles should differ from the properties of three known generations. The bound from the invisible Z decay width forbids further light generations, especially additional neutrinos. The existing experiments give lower bounds on masses of fourth generation leptons (charged lepton and neutrinos) at the level of 100 GeV and an upper bound on the mixing parameters $0.04 - 0.08$.

The generation of neutrino mass in models with four fermion generations has been explored in several publications. The simplest model with four SM generations and usual massless neutrinos at tree level has one right-handed (RH) neutrino [45]. An explanation of neutrino masses in terms of the usual see-saw mechanism [46], however, requires at least three RH neutrinos. Since the fourth generation neutrino should be much heavier than the three SM neutrinos, several authors suggested its pseudo-Dirac nature [45; 47; 48]. The light neutrino masses can be generated by two loop diagrams with two W bosons exchange in the

framework of five SM generations [49]. There are studies of the leptonic flavour structure in SM4 with discrete flavour symmetries $\mathbb{Z}_4$ [50] and A_5 [51]. Moreover, the leptonic flavour structure have been explored in extra-dimensional 4 generation models [52].

The origin of neutrino masses or the flavour structure, and especially the number of chiral SM generations are not understood within the Standard Model.

In this paper, we present a comprehensive study of the neutrino mass generation in the SM model with four fermionic generations including one RH neutrino per generation. We will restrict ourselves to a non-SUSY model. However, our results can be directly generalised to the SUSY case. Besides the usual see-saw contribution, we calculate and study the contributions from tree level mechanism related to mixing of the fourth neutrino with the light ones, and from the two $W-$boson exchange at two loop. We explore possible flavour symmetries in the context of SM4. We study the smallest discrete group with a four-dimensional representation and explore flavor structures that appear in the most economical scenarios.

The paper is organised as follows. The contributions to the neutrino mass matrix from three different mechanisms are computed in Sec. 2. In Sec. 3, we consider existing bounds on fourth generation leptons, in particular, from the neutrinoless double beta decay. We find the regions in parameter space in which different mechanisms dominate. We explore possible realisations of flavour symmetries in context of SM4 in Sec. 4, and conclude in Sec. 5. In the Appendix we present the group theoretical details of the smallest group with a representation $\underline{\mathbf{4}}$: $\mathrm{SG}(20,3) \cong \mathbb{Z}_5 \rtimes_\varphi \mathbb{Z}_4$.

2 Contributions to Neutrino Masses

We consider the extension of the SM by one RH singlet fermion per generation, N_k, $k = 1,2,3$ for the first three generations and N_4 for the fourth generation. We consider two SM Higgs doublets, one coupling to neutrinos H_1 and one to charged leptons H_2. The case with a single Higgs doublet is obtained by identifying $H = H_1$ and $H^C = H_2$. In the flavour basis, the leptons have the following couplings

$$-\mathcal{L} = Y_{\alpha k}\bar{\ell}_\alpha H_1 N_k + Y_{\alpha 4}\bar{\ell}_\alpha H_1 N_4 + Y_{Ek}\bar{\ell}_E H_2 N_k + Y_{E4}\bar{\ell}_E H_2 N_4 + \frac{1}{2}M_k N_k^T N_k + \frac{1}{2}M_4 N_4^T N_4 + \text{h.c.}, \tag{1}$$

where ℓ_α, $\alpha = e, \mu, \tau$ and ℓ_E denote the light and the fourth generation left-handed lepton doublets, respectively. These couplings lead to the following neutral fermion mass matrix in the flavour basis $\begin{pmatrix}\nu_\alpha & \nu_E & N_4 & N_k\end{pmatrix}^T$

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & f_L & m \\ \dots & 0 & m_{E4} & f_R^T \\ \dots & \dots & M_4 & 0 \\ \dots & \dots & \dots & M \end{pmatrix} . \tag{2}$$

We take the RH neutrino mass matrix to be diagonal: $M = \mathrm{diag}(M_1,\, M_2,\, M_3)$. The complete Dirac mass matrix consists of the following components:

- the Dirac mass matrix of the light SM neutrinos, $m_{\alpha k} = Y_{\alpha k} v_{\rm EW}^{\nu}$ ($\alpha = e,\, \mu,\, \tau$ and $k = 1, 2, 3$) with $v_{\rm EW}^{\nu} = \langle H_1 \rangle$ being the vacuum expectation value (VEV) of the Higgs H_1 coupling to neutrinos,

- the Dirac mass of the fourth generation: $m_{E4} = Y_{E4} v_{\rm EW}^{\nu}$, and

- the column f_L which gives mixing of the fourth generation with the light ones $f_{L\alpha} = Y_{\alpha 4} v_{\rm EW}^{\nu}$, and $f_{Rk} = Y_{Ek} v_{\rm EW}^{\nu}$.

The neutrino Dirac mass can be written as

$$U_L \,\mathrm{diag}\,(m_i)\; U_R^{\dagger}\,. \tag{3}$$

According to (2) the left-handed (LH) mixing matrix elements of the fourth generation with the three light generations are given approximately by

$$(U_L)_{\alpha 4} \simeq \frac{f_{L\alpha}}{m_{E4}}, \tag{4}$$

which are bounded by experiments to be smaller than $0.04 - 0.08$ [53]. Similarly, the RH mixing matrix elements can be estimated as

$$(U_R)_{k4} \simeq \frac{f_{Rk}}{m_{E4}}\,, \tag{5}$$

provided that m_{E4} dominates the Dirac mass matrix.

Decoupling of the RH neutrinos, $N_{1,2,3}$, in Eq. (2) leads to the effective mass matrix in the basis $\begin{pmatrix}\nu_\alpha, & \nu_E, & N_4\end{pmatrix}^T$:

$$\mathcal{M}' = \begin{pmatrix} -mM^{-1}m^T & -mM^{-1}f_R & f_L \\ ... & -f_R^T M^{-1} f_R & m_{E4} \\ ... & ... & M_4 \end{pmatrix}\,. \tag{6}$$

At this level the three active neutrinos acquire the usual see-saw contributions associated to the three heavy RH neutrinos. Notice that in the limit $f_R = 0$ further decoupling of ν_E and N_4 in Eq. (6) gives zero contribution to the light neutrino masses in spite of the fact that ν_α interacts with N_4.

2.1 Tree-level Mechanism from Mixing with Fourth Generation

Depending on value of M_4 there are two extreme cases for the masses of the fourth neutrino: the see-saw case, $M_4 \gg m_{E4}$ and the pseudo-Dirac case $M_4 \ll m_{E4}$.

1). In the see-saw case after decoupling the fourth RH neutrino we obtain the 4×4 neutral fermion mass matrix in the basis (ν_α, ν_E):

$$\begin{pmatrix} -mM^{-1}m^T - \frac{f_L^T f_L}{M_4} & -\frac{m_{E4}}{M_4} f_L \\ \dots & -\frac{m_{E4}^2}{M_4} \end{pmatrix} \tag{7}$$

where we have neglected the see-saw contributions of the first three RH neutrinos to the fourth row and column since $M_4 \ll M_k$. The mixing matrix elements can be estimated as

$$U_{\alpha 4} \simeq \frac{f_{L\alpha}}{m_{E4}} \simeq (U_L)_{\alpha 4} \, . \tag{8}$$

Further decoupling of ν_4 leads to cancellation of the leading order contribution of the fourth generation to the light 3×3 neutrino mass matrix. Non-zero contributions are generated by the next-to-leading order see-saw effect [54]. Therefore, in this limit, a fourth generation does not give a substantial tree-level contribution to the light neutrino mass.

2) In the pseudo-Dirac case, $M_4 \ll m_{E4}$, under the assumption of $f_{Rk} \ll m_{E4}$ a block diagonalisation of matrix (6) (i.e. decoupling the pseudo-Dirac pair $(\nu_4,\, N_4)$) leads to

$$m_\nu \simeq m^{\rm ss} + m^{\rm tree} \tag{9}$$

with $m^{\rm ss} \equiv -mM^{-1}m^T$ and

$$m^{\rm tree} = \frac{1}{m_{E4}} \left[\left(mM^{-1} f_R f_L^T \right) + (...)^T \right] . \tag{10}$$

The contribution m^{tree} is linear in the light Dirac mass m and therefore can be considered as a new realisation of the linear see-saw [55]. Up to high order corrections the total mass matrix in Eq. (9) can be rewritten as

$$m_\nu = - \left(m - \frac{1}{m_{E4}} f_L f_R^T \right) \left(M + \frac{M_4}{m_{E4}^2} f_R f_R^T \right)^{-1} \left(m - \frac{1}{m_{E4}} f_L f_R^T \right)^T \, . \tag{11}$$

Thus the total mass matrix can be considered either as a combination of linear and ordinary type-I see-saw (as in Eq. (9)) or as a type-I see-saw with a modified Dirac neutrino mass term.

We can rewrite the contribution of the fourth generation to the $\alpha\beta$ matrix element of m_ν as

$$m_{\alpha\beta}^{\rm tree} \simeq m_{E4} \sum_k \frac{m_{\alpha k}}{M_k} (U_L)_{\beta 4} (U_R)_{k4} + (\alpha \leftrightarrow \beta) \, . \tag{12}$$

Hence, $m_{\alpha\beta}^{\rm tree}$ is suppressed by the left- and right-handed mixing in addition to the usual see-saw type factor. As the leading contribution has rank one and the sub-leading ones are suppressed, it can only generate one mass scale and the ordinary see-saw contribution

cannot be completely neglected. We can compare the contributions of the fourth generation via the $k^{\rm th}$ RH neutrino with the see-saw contribution of the $k^{\rm th}$ RH neutrino to the light neutrino mass matrix as

$$\frac{(m_k^{\rm tree})_{\alpha\beta}}{(m_k^{\rm ss})_{\alpha\beta}} \simeq \frac{(U_R)_{k4}(U_L)_{\alpha 4}}{m_{k\beta}/m_{E4}} + (\alpha \leftrightarrow \beta)\ . \tag{13}$$

Therefore the tree level contributions via the $k^{\rm th}$ RH neutrino dominates over the seesaw if

$$(U_R)_{k4}(U_L)_{\alpha 4} \gtrsim m_{k\beta}/m_{E4}. \tag{14}$$

2.2 Two-Loop Mechanism

If the components of the fourth generation neutrino are Majorana particles, they induce a Majorana mass term for the light neutrinos at two loop level, $m^{\rm loop}$, see Fig. 1(a) [56]. We will consider the system of five neutrinos (ν_α, ν_E, N_4) after decoupling of the heavy RH neutrino components. The tree level mass matrix is then given by Eq. (6) and we neglect the see-saw contributions to the $\nu_\alpha - \nu_E$ as well as $\nu_E - \nu_E$ elements.

The expression for the two-loop generated Majorana masses given in Eqs. (21,22) of [56] can be rewritten in the flavour basis as

$$m_{AB}^{\rm loop} \simeq -\frac{g^4}{m_W^4} m_{E4}^2 M_4 m_A^2 m_B^2 (U_L)_{A4}(U_L)_{B4} I_{AB}\ . \tag{15}$$

Here the indices $A, B = e, \mu, \tau, E$ run over four generations, m_A and m_B are the charged lepton masses, g is the SU(2) gauge coupling and m_W is the mass of the W-boson. The integral I_{AB} equals

$$I_{AB} = \int \frac{{\rm d}^4 p}{(2\pi)^4} \int \frac{{\rm d}^4 q}{(2\pi)^4} \frac{p\cdot q}{(p^2-m_A^2)(q^2-m_B^2)} \frac{1}{(p+q)^2-m_{N1}^2} \frac{1}{(p+q)^2-m_{N2}^2} \left[\frac{1}{p^2q^2} - \frac{3}{4}\frac{1}{p^2-m_W^2}\frac{1}{q^2-m_W^2}\right]\ . \tag{16}$$

Here $m_{N1,2}$ are the eigenvalues of the mass matrix of the fourth generation neutrino states. According to Eq. (15), the two loop generated masses depend on the mixing of the fourth generation neutrino with the light neutrinos, U_{A4}, the Dirac mass of the fourth generation neutrino, m_{E4}, and the Majorana mass of the RH neutrino, M_4.

There are two different two-loop contributions to the mass matrix of light neutrinos: (i) the direct one which follows from Eq. (15) for $A, B = e, \mu, \tau$, and (ii) the contribution via the EE-element, the Majorana mass of ν_E, m_{EE} generated in 2 loops. As we will see, the latter dominates due to hierarchy of the charged lepton masses: $m_E \gg m_{e,\mu,\tau}$. In fact, this contribution can be computed in the approximation of vanishing charged lepton masses $m_{e,\mu,\tau} = 0$.

Let us compute the second contribution in the pseudo-Dirac case when the masses of neutrinos of the fourth generation equal $m_{N1} \simeq m_{N2} \simeq m_4$ and the splitting between them given by M_4 is small: $M_4 \ll m_4$. According to Eq. (15) the m_{EE} element is given by

$$m_{EE}^{\rm loop} \simeq -g^4 m_W^{-4} m_4^2 M_4 m_E^4 (U_L)_{E4}^2 I_{EE} \; . \tag{17}$$

In the limit $m_W \ll m_4 \ll m_E$ the integral I_{EE} equals approximately *

$$I_{EE} \simeq \frac{1}{(4\pi)^4 m_E^2}\left(\frac{\pi^2}{3} - 2 + \ln\frac{m_4^2}{m_E^2}\right) \; . \tag{18}$$

Now we have the mass matrix (6) with non-zero elements in the fifth row and column and non-zero EE-element. Decoupling of the fourth (pseudo-Dirac) neutrino (*i.e.* the see-saw diagonalisation with ν_E, N_4 heavy block) contributes to the masses of the light neutrinos:

$$m^{\rm loop} \simeq m_{EE}^{\rm loop} \frac{f_L^T f_L}{m_{E4}^2 - m_{EE}^{\rm loop} M_4} \; , \tag{19}$$

which in the case $m_{E4}^2 \gg m_{EE}^{\rm loop} M_4$ leads to

$$m_{\alpha\beta}^{\rm loop} \simeq m_{EE}^{\rm loop} \frac{f_{L\alpha}}{m_{E4}} \frac{f_{L\beta}}{m_{E4}} = m_{EE}^{\rm loop} (U_L)_{\alpha 4} (U_L)_{\beta 4} \; . \tag{20}$$

The resulting contribution to the light neutrino mass matrix is

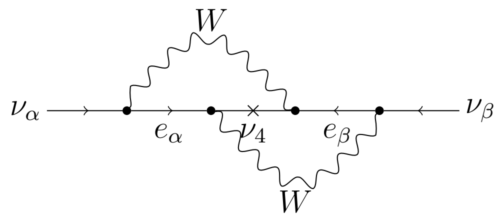
$$\begin{aligned} m_{\alpha\beta}^{\rm loop} &\simeq -\frac{g^4 m_E^4}{m_W^4} M_4 m_4^2 I_{EE} \, (U_L)_{E4}^2 \, (U_L)_{\alpha 4} \, (U_L)_{\beta 4} \\ &= -\frac{g^4 \, (U_L)_{E4}^2}{(4\pi)^4} \left(\frac{\pi^2}{3} - 2 + \ln\frac{m_4^2}{m_E^2}\right) \frac{m_4^2 m_E^2}{m_W^4} M_4 \, (U_L)_{\alpha 4} \, (U_L)_{\beta 4} \; . \end{aligned} \tag{21}$$

The mass matrix formed by the loop contribution via $m_{EE}^{\rm loop}$ (21) is singular (rank 1) with the unique non-zero eigenvalue

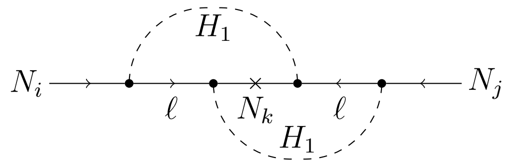
$$m_4^{\rm loop} \approx \frac{g^4 m_E^4}{m_W^4} M_4 m_4^2 \left| I_{EE} \sum_\alpha (U_L)_{\alpha 4}^2 \right| \; . \tag{22}$$

When the masses of three SM charged lepton are taken into account, the two loop contribution obtains full rank. However due to a strong hierarchy of these masses the two loop contribution cannot explain neutrino masses by themselves [49]. The contribution (21) dominates over the direct two loop contribution (15) due to three known SM leptons. Indeed, the ratio of the two equals:

$$\frac{m_\alpha^2 m_\beta^2}{m_E^4} \frac{I_{\alpha\beta}}{I_{EE}}. \tag{23}$$



(a) Two W exchange contribution to light neutrino mass matrix.



(b) Rank changing two loop diagram contributing to Majorana mass term.

Figure 1: The two loop diagrams which describe contributions to Majorana neutrino masses. Shown are the diagrams for the light neutrinos (left), and for the heavy RH neutrino (right).

The flavour structure of the loop contribution is determined by the mixing matrix elements $(U_L)_{\alpha 4}$. Note that the results considered here can be immediately obtained by computing diagrams with the would-be Goldstone bosons, then all the substantial quantities arise from vertices.

In the see-saw case for the fourth generation masses, $M_4 \gg m_4$ the two-loop contribution is suppressed by $f_{L\alpha}/m_{E4}$ only, because the direct Majorana mass term M_4 is large. Consequently, the mixing between the fourth generation and the three light SM generation has to be small.

2.3 Radiative Generation of Fourth Generation Singlet Majorana Mass

Similarly to m_{EE} considered in the previous subsection, the RH Majorana neutrino mass M_4 can be generated at the two loop level. The relevant (rank changing) two loop diagram is shown in Fig. 1(b), which results in the following expression for the mass in the $\overline{\text{MS}}$ renormalisation scheme

$$M_{ij}^{\text{loop}} = \frac{2}{(16\pi^2)^2} \sum_{k=1}^{4} (Y^\dagger Y)_{ik} \left(Y^\dagger Y\right)_{jk} M_k \left(\frac{1}{\epsilon} + \frac{1}{2} + \ln\frac{\mu^2}{M_k^2}\right) . \tag{24}$$

Here $i, j = 1, \ldots, 4$ and the Higgs mass has been neglected. The RG produced fourth generation Majorana mass can be estimated (neglecting diagrams with light charged fermions) as

$$M_4^{\text{loop}} \simeq \frac{y_4^4}{(8\pi^2)^2} \sum_{i=1}^{3} \left[(U_R)_{i4}^*(U_R)_{E4}\right]^2 M_i \ln\frac{M_i}{\Lambda} \tag{25}$$

with Λ being the high scale, at which the theory is defined, and $y_4 = m_{E4}/v_{\text{EW}}^\nu$ being the neutrino Yukawa coupling of the fourth generation, which dominates over the other Yukawa

*See the appendix of [49] for the evaluation of this integral.

couplings. An estimate can also been be found in [57]. In addition to the RG running, the finite part of the counter term leads to a scheme dependent threshold correction [†]. The main contribution comes from the diagrams with the charged lepton E. According to Eq. (25) $M_4^{\rm loop} \propto m_4^4 U_R^2$ and therefore it quickly decreases with m_4. For $U_R \sim U_L$ inspired by the L-R symmetry and a single Higgs, *i.e.* $H_1 = H$ and $H_2 = H^C$, the mass can be estimated as $M_4 = 1.0\,\mathrm{GeV}$ for $U_R = 0.001$, $m_4 = 400\,\mathrm{GeV}$, $M_i = 10^8\,\mathrm{GeV}$ and $\Lambda = 10\,M_i$.

Neglecting an accidental cancellation, we expect that the fourth generation Majorana mass, M_4, has at least the size of the radiatively generated contribution given in Eq. (25): $M_4 \geq M_4^{\rm loop}$. Using $M_4^{\rm loop}$ only, we can estimate the size of the contributions to the neutrino masses using Eq. (21):

$$m_{\alpha\beta}^{\rm loop} \simeq -\frac{y_4^4 g^4 \left(U_L\right)_{E4}^2}{4(8\pi^2)^4} \left(\frac{\pi^2}{3} - 2 + \ln\frac{m_4^2}{m_E^2}\right) \frac{m_4^2 m_E^2}{m_W^4} \left(U_L\right)_{\alpha 4} \left(U_L\right)_{\beta 4} \sum_{i=1}^{3} \left[(U_R)_{i4}^* \left(U_R\right)_{E4}\right]^2 M_i \ln\frac{M_i}{\Lambda} \,. \tag{26}$$

Effectively it is generated at four loops level. According to Eq. (26), and since $(U_L)_{E4} \approx (U_R)_{E4} \approx 1$ the bound on the neutrino mass scale leads to an upper bound on the combination $|(U_R)_{i4}(U_L)_{\alpha 4}|^2 M_i$. The bound strongly depends on $m_4 = y_4 v_{\rm EW}^{\nu}$ and m_E.

Fig. 2 shows the iso-contours of the two loop contribution to the light neutrino masses in the plane of mixing angle $|U_{\alpha 4}|$ and the fourth generation Majorana mass M_4. The equation for these contours is given by Eq. (21) which can be rewritten as

$$|(U_L)_{\alpha 4}| = \sqrt{\left|\frac{m^{\rm loop}}{C_2 M_4}\right| \frac{m_W^2}{m_E m_4}} \,, \tag{27}$$

where

$$C_2 \equiv -\frac{g^4 \left(U_L\right)_{E4}^2}{(4\pi)^4} \left(\frac{\pi^2}{3} - 2 + \ln\frac{m_4^2}{m_E^2}\right) \,. \tag{28}$$

The vertical lines in Fig. 2 indicate the Majorana mass M_4 generated by the two loop correction (25) for $y_4 = m_4/v_{\rm EW}^{\nu}$. So, $|(U_L)_{\alpha 4}| \propto 1/\sqrt{M_4}$, and furthermore the mixing becomes small with increase of m_E. Values of mixing parameters $(U_L)_{\alpha 4}$ at the level achievable by the direct searches can be obtained only for very small RH neutrino masses: $M_4 < 1$ keV. On the other hand M_4 of the size ~ 1 GeV requires $(U_L)_{\alpha 4} \sim 10^{-5}$.

The bound on the light neutrino mass scale strongly constrains the combination $|(U_L)_{\alpha 4}^2\, M_4|$. Barring accidental cancellations, we expect the fourth generation RH neutrino mass to be at least of the scale generated by the two loop diagrams. Under this assumption, the mixing parameter $|U_{\alpha 4}|$ should be smaller than the value at the intersection of the vertical line with the lower border of the excluded shaded area.

[†]Notice that this result for the radiatively induced RH neutrino mass is rather general and valid beyond the four generation context. It is particularly interesting in case of the nearly singular see-saw when one of the mass eigenvalues is considerably smaller than the remaining ones.

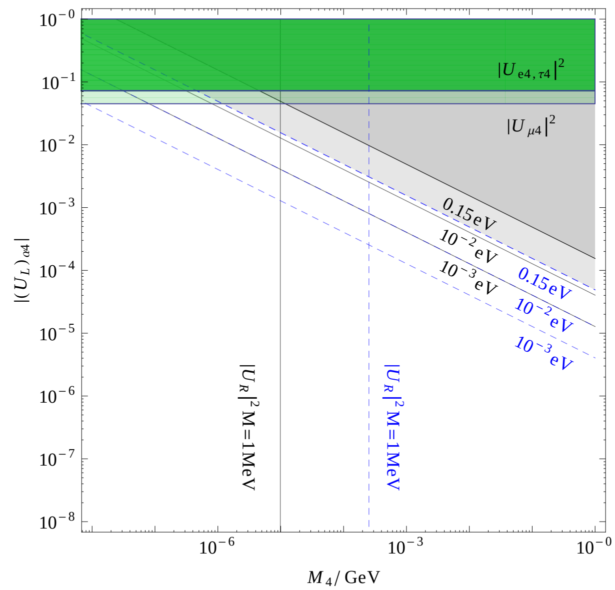


Figure 2: Iso-contours of the two loop contribution to light neutrino masses in the $(U_L)_{\alpha 4} - M_4$ plane for different values of the Dirac neutrino mass, m_4, and charged lepton mass, m_E. The black solid lines correspond to $m_4 = 400\,\mathrm{GeV}$ and $m_E = 600\,\mathrm{GeV}$, while the blue dashed lines correspond to $m_4 = 900\,\mathrm{GeV}$ and $m_E = 1\,\mathrm{TeV}$. The shaded region is excluded by the cosmological bounds on the neutrino mass. The green shaded areas are excluded by the bounds on the mixing angles $U_{\alpha 4}$ (see Sec. 3.1). The vertical lines correspond to the Majorana mass M_4 induced at the two loops level for a RH neutrino mixing $|U_R| = 10^{-4}$ and RH neutrino mass $M = 100\,\mathrm{TeV}$ with the cutoff scale $\Lambda = 1000\,\mathrm{TeV}$.

These two loop corrections do not exist in SUSY due to the non-renormalisation theorem for the superpotential [58]. However, if there are RH neutrinos with a mass below the SUSY breaking scale, there are quantum corrections. The larger particle content in the SUSY version compensates for the smaller logarithms coming from the RG corrections (see *e.g.* [59] for two loop corrections to the light neutrino mass matrix).

3 Phenomenology of Fourth Generation Neutrinos

3.1 Existing Bounds on Fourth Generation Leptons

Let us summarise the bounds on masses and mixing of the fourth generation particles which we will use in our analysis. The collider searches give [60]

$$m_E > 100.8\,\mathrm{GeV}\,, \quad m_N > (80.5 - 101.5)\,\mathrm{GeV}\,. \tag{29}$$

The range of values for the lower bounds on m_N in Eq. (29) originates from different search channels, $N \to W^* + (e,\, \mu,\, \tau)$ under the assumption of a 100% branching ratio in a given channel. The bounds depend also on the nature of the neutrino: for Majorana neutrino they are about 10 GeV weaker than for Dirac neutrinos. The bounds rely on the assumption that only one heavy neutral lepton can be produced. A recent reanalysis [36] shows that the bounds can be relaxed when two heavy neutral leptons are accessible, like in the framework of pseudo-Dirac neutrinos. Under the assumption of a mass splitting $M_{N_2} - M_{N_1} > 10\,\mathrm{GeV}$ between the two heavy neutral states $N_{1,2}$, the study of $e^+e^- \to Z^* \to N_i N_j \to lW^* lW^* Z^{*0,1,2}$ leads to the bounds $62.1\,\mathrm{GeV}(W^*\tau)$, $79.9\,\mathrm{GeV}(W^*\mu)$ and $81.8\,\mathrm{GeV}(W^*e)$. The number of Z^*-bosons depends on the number of produced N_2 via $N_2 \to N_1 Z^*$. In our study, we mainly consider smaller mass splittings $M_{N_2} - M_{N_1}$, where the branching ratio of $N_2 \to lW^*$ dominates over the one of $N_2 \to N_1 Z^*$. This leads to an interference between $N_{1,2} \to lW^*$ and we expect the bounds to become weaker.

A study of the sensitivity of the Tevatron to a fourth generation neutrino [37] shows that it can put a lower bound $m_N > 175\,\mathrm{GeV}$ and has a 3σ discovery potential for $m_N < 150\,\mathrm{GeV}$ with 5 fb^{-1}. The LHC can exclude fourth generation charged leptons up to 250 GeV [35].

The electroweak precision tests (which include quark mixing but neglect leptonic mixing) constrain the mass splitting between the fourth generation leptons [4]

$$|m_E - m_N| < 140\,\mathrm{GeV}, \tag{30}$$

indicating that the masses of the fourth generation leptons should be of the same order of magnitude.

The leptonic mixing angles are constrained by searches for the radiative μ^- and τ^- decays, $\ell_i \to \ell_j \gamma$, as well as by kaon and pion decays. The limits given in [53] read

$$U_{\mathrm{PMNS}} = \begin{pmatrix} . & . & . & < 0.073 \\ . & . & . & < 0.045 \\ . & . & . & < 0.072 \\ < 0.092 & < 0.092 & < 0.092 & > 0.9958 \end{pmatrix}. \tag{31}$$

There is an even stronger bound on $U^*_{\mu 4} U_{e4}$ from the $\mu - e$ conversion: $|U^*_{\mu 4} U_{e4}| < 0.4 \cdot 10^{-4}$ for $m_N > 100$ GeV [61].

The influence of mixing of light generations on the masses of the fourth neutrino can be neglected. We can estimate the maximal allowed value of M_4 which is realized in the see-saw limit, $M_4 \gg m_4$, as $M_4 = m_4^2/m_N$. Using the unitarity upper limit on $m_4 \lesssim 1.2\,\mathrm{TeV}$ [20] and the LEP exclusion limit for an additional neutral lepton $m_N \sim 100\,\mathrm{GeV}$ we find

$$M_4 \lesssim 14\,\mathrm{TeV}. \tag{32}$$

3.2 Neutrinoless double beta decay and Cosmological Bounds

All three main mechanisms of light neutrino mass generation are essentially of the see-saw type and the $\beta\beta_{0\nu}$-decay proceeds via the neutrino exchange only. Therefore, we can apply here the results of [62]. Following [62], we separate the contributions to the amplitude of the decay from a heavy mass eigenstates with a mass $m_I \gg m_\pi \sim 100\,\mathrm{MeV}$, and from the light neutrino mass eigenstates with mass $m_i \ll m_\pi$:

$$A \propto \sum_i^{\text{light}} m_i U_{ei}^2 M^{0\nu\beta\beta}(m_i) + \sum_I^{\text{heavy}} m_I U_{eI}^2 M^{0\nu\beta\beta}(m_I) \ , \tag{33}$$

where the masses $m_{i,I}$ and mixing angles U_{ei}, U_{eI} are defined by

$$U^* \mathrm{diag}(m_1, \ldots, m_8) U^\dagger = \mathcal{M} \tag{34}$$

with $\mathcal{M}$ being the 8×8 neutral fermion mass matrix.

The two loop direct contribution to the m_{ee} element in the flavour basis is negligible being proportional to m_e^4. Therefore, according to Eq. (2) $m_{ee} \approx 0$. In terms mixing angles and mass eigenstates defined in Eq. (34) this condition can be expressed as

$$\sum_i^{\text{light}} m_i U_{ei}^2 + \sum_I^{\text{heavy}} m_I U_{eI}^2 \approx 0 \ . \tag{35}$$

The nuclear matrix elements $M^{0\nu\beta\beta}$ in Eq. (33) include neutrino propagators:

$$D_\nu \propto \begin{cases} \frac{1}{p^2 - m_i^2} \approx \frac{1}{p^2}, & \text{for} \quad m_i \ll m_\pi \\ \frac{1}{p^2 - m_I^2} \approx -\frac{1}{m_I^2}, & \text{for} \quad m_I \gg m_\pi \end{cases} , \tag{36}$$

where $m_\pi \sim 1/r_N$ is the pion mass, which gives the inverse size of the nucleus radius, r_N. Therefore $M^{0\nu\beta\beta}$ practically does not depend on the mass of the exchanged light neutrinos: $M^{0\nu\beta\beta}(m_i) \approx M^{0\nu\beta\beta}(0)$. For heavy neutrinos the matrix element decreases as $M^{0\nu\beta\beta}(m_I) \propto m_I^{-2}$. Consequently, the ratio of the matrix elements

$$\frac{M^{0\nu\beta\beta}(m_I)}{M^{0\nu\beta\beta}(m_i)} \sim \frac{m_\pi^2}{m_I^2} \ll 1. \tag{37}$$

Using relation (35) we can rewrite the amplitude Eq. (33) in the following way

$$\begin{aligned} A &\propto \sum_i^{\text{light}} m_i U_{ei}^2 M^{0\nu\beta\beta}(0) + \sum_I^{\text{heavy}} m_I U_{eI}^2 M^{0\nu\beta\beta}(m_I) \\ &= \sum_I^{\text{heavy}} m_I U_{eI}^2 (M^{0\nu\beta\beta}(m_I) - M^{0\nu\beta\beta}(0)) \\ &\approx -\sum_I^{\text{heavy}} m_I U_{eI}^2 M^{0\nu\beta\beta}(0) = \sum_i^{\text{light}} m_i U_{ei}^2 M^{0\nu\beta\beta}(0) \ , \end{aligned} \tag{38}$$

where we used $M^{0\nu\beta\beta}(m_I) \ll M^{0\nu\beta\beta}(0) \approx M^{0\nu\beta\beta}(m_i)$ in the first and third line and Eq. (35) in lines two and three. Hence, the dominant contribution to $\beta\beta_{0\nu}$-decay is from light neutrinos.

Thus, the bound from $\beta\beta_{0\nu}$-decay is reduced to the bound on the effective Majorana mass of the electron neutrino due to light neutrinos only. That is, the $\beta\beta_{0\nu}$-decay gives a bound on the light neutrino masses, which has also been pointed out in [49], and as far as this bound is satisfied, no other bounds on the model appear. Hence, $\beta\beta_{0\nu}$-decay restricts the model via the light masses only.

Notice that in the discussion of $\beta\beta_{0\nu}$-decay in [63], the light contribution has been neglected.

At the moment, cosmology gives even a stronger bound on light neutrino masses than the $\beta\beta_{0\nu}$-decay. We took $m_0 \lesssim 0.15\,\mathrm{eV}$ for an individual neutrino as reference value which originates from the bound $\sum m_i \lesssim 0.44$ eV [64].

3.3 Comparison of Different Contributions to Neutrino Mass

As we have found in the previous section, in models with four families of fermions, generically there are three contributions to the light neutrino masses from three different mechanisms: (i) the usual see-saw type-I, m^{ss}; (ii) the tree-level contribution m^{tree} due to mixing of the light neutrinos with ν_4 is essentially another see-saw, it is linear in the usual Dirac mass matrix; (iii) the 2-loop contribution induced by the Majorana mass term of the neutrino of the fourth family, m^{loop}. These three contributions have different flavour structures but partially correlate. For a given Dirac mass matrix of light neutrinos: $m^{\mathrm{ss}} = m^{\mathrm{ss}}(M_k)$, $m^{\mathrm{tree}} = m^{\mathrm{tree}}(M_k, U_R, U_L)$ (Eq. (12)), and $m^{\mathrm{loop}} = m^{\mathrm{loop}}(M_k, U_R, U_L)$ (Eq. (21)).

In what follows, we will consider these contributions in the case of a vanishing fourth generation Majorana mass term M_4 at tree-level. The mass M_4 is constrained on the one hand by the invisible Z-decay width and on the other hand by the bound on the neutrino mass which is induced at two loop (see Fig. 2). Furthermore, we restrict ourselves to a single Higgs doublet, *i.e.* $H_1 = H$ and $H_2 = H^C$. Similar conclusions apply in a two Higgs doublet model. The main difference is an increased neutrino Yukawa coupling, for fixed values of masses. This leads to a larger loop contribution to neutrino masses. The following discussion does not depend on the Higgs mass m_H, as long as it is negligible compared to the heavy RH neutrino masses M_i, $i = 1, 2, 3$. If they are of a similar magnitude, the expression (25) for M_4 will change, but it remains valid as an order of magnitude estimate. Hence, the following conclusions are also valid for a heavy SM Higgs $m_H \gtrsim 600\,\mathrm{GeV}$.

We will first discuss the "1+1" generation case: one light generation and the fourth generation [‡]. In this case we have one light neutrino with Dirac mass $m \equiv m_{\alpha i}$ and one very heavy RH neutrino with mass $M \equiv M_i$. We can introduce a single parameter which

[‡]The results can be directly applied to one specific matrix element in case of 3+1 generations.

characterises mixing of the light neutrino with the neutrino of the fourth generation:

$$\xi \equiv (U_L)_{\alpha 4}(U_R)_{i4}. \tag{39}$$

In terms of this parameter the tree level contribution (12) can be written as

$$m^{\rm tree} = m_{E4}\frac{2m}{M}\xi. \tag{40}$$

The loop contribution (26) is then

$$m^{\rm loop} = C^{\rm loop} m_E^2 m_4^6 \xi^2 M \ln\frac{M}{\Lambda}\ , \tag{41}$$

where

$$C^{\rm loop} \equiv -\frac{g^4}{4(8\pi^2)^4 m_W^4}\frac{1}{{v_{\rm EW}^{\nu}}^4}\left(\frac{\pi^2}{3} - 2 + \ln\frac{m_4^2}{m_E^2}\right) \tag{42}$$

and we have taken into account that $(U_L)_{E4} \approx (U_R)_{E4} \approx 1$. Let us underline that mixing parameters of the fourth neutrino enter the contributions only in the combination ξ. The other relevant parameters are m_E, m_4, M and m. Relative contributions of different mechanisms depend on values of these parameters.

In Fig. 3 we show the iso-contours of different contributions to neutrino mass in the $\xi - M$ plane for fixed values of m_E, m_4 and m. The equations for these contours can be readily obtained from Eq. (40) and Eq. (41). For a given value of $m^{\rm tree}$ we find from Eq. (40) the following dependence of ξ on M:

$$\xi^{\rm tree} = M\frac{m^{\rm tree}}{2mm_{E4}}. \tag{43}$$

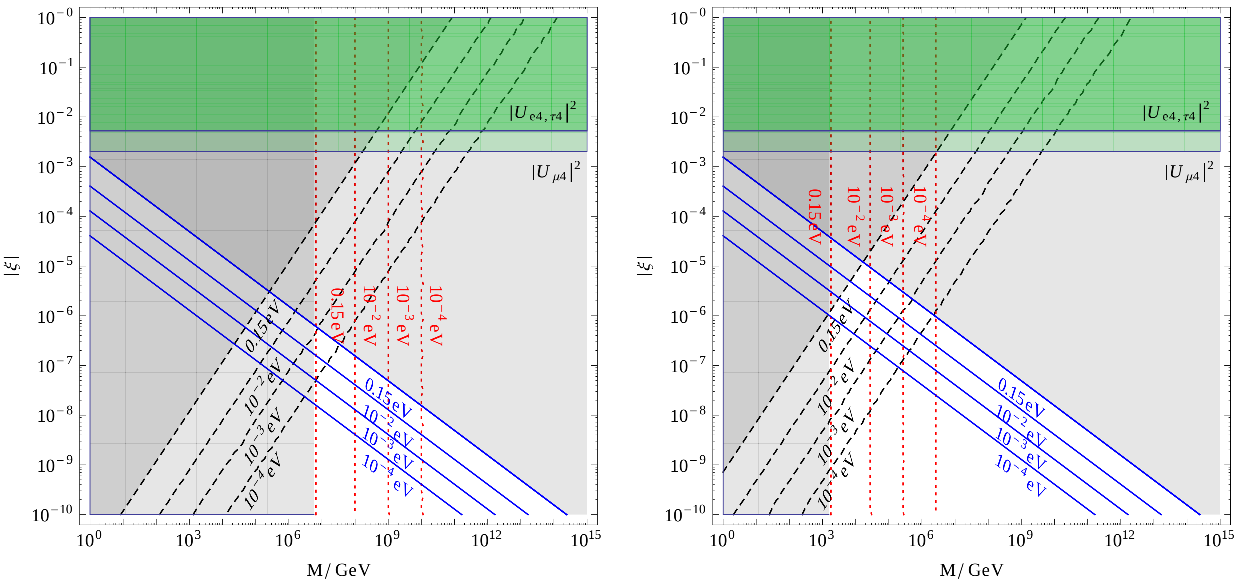
That is, $\xi^{\rm tree}$ linearly increases with M; it is proportional to $m^{\rm tree}$ and inversely proportional to m. The iso-contours of the tree-level contribution correspond to the black dashed lines.

From Eq. (41) we obtain the analytic expression for iso-contours of the loop contribution (blue solid lines in Fig. 3):

$$\xi^{\rm loop} = \left[\frac{m^{\rm loop}}{C^{\rm loop} m_E^2 m_4^6}\right]^{1/2}\cdot\frac{1}{\sqrt{M\ln\frac{M}{\Lambda}}}\ . \tag{44}$$

For $M \propto \Lambda$ this equation gives $\xi^{\rm loop} \propto 1/\sqrt{M}$.

The usual see-saw contribution, $m^{\rm ss} = -m^2/M$, does not depend on ξ and the corresponding iso-contours are just vertical red dotted lines in the plot of Fig. 3. According to Fig. 3, the loop contribution dominated for large values of M and small values of ξ. The tree level contribution is larger for small M and large ξ, whereas the usual see-saw dominates in the range of small M. For $m = 31\,\mathrm{MeV}$ (see Fig. 3(a)) the allowed region is $M \gtrsim 10^7$



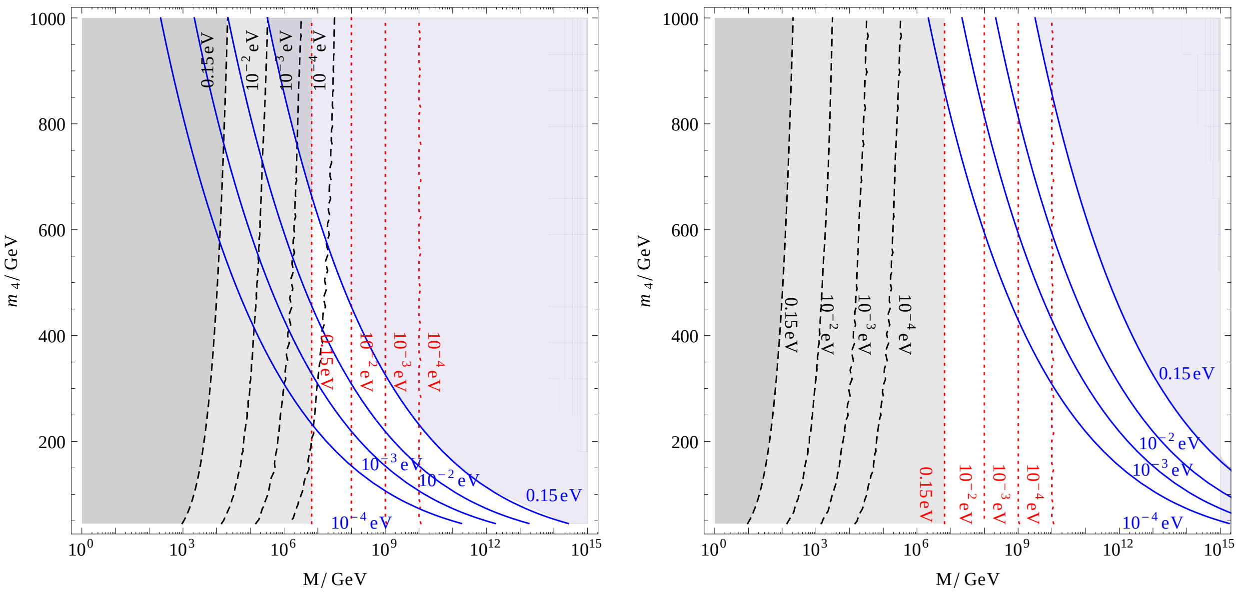
(a) $m = 31\,\mathrm{MeV}$ (b) $m = m_e$

Figure 3: Iso-contours of different contributions to light neutrino masses (numbers at the curves) from different mechanisms in the $\xi - M$ plane for $m_4 = 400\,\mathrm{GeV}$ and $m_E = 600\,\mathrm{GeV}$ and two different values of the Dirac mass m. Black dashed lines correspond to the tree-level contribution, blue solid lines to the loop contribution and red dotted ones to the usual see-saw contribution. Note that the Higgs mass m_H has been neglected in the calculation of the radiatively induced M_4. Hence for $M \sim m_H$, the loop contribution can only be considered as an order of magnitude estimate.

GeV and $\xi \lesssim 10^{-6}$. The tree-level contribution is negligible in this region and the total neutrino mass is determined by an interplay of the usual see-saw and the loop contributions. Furthermore, the see-saw dominates at smaller M and ξ.

With decrease of m, the relative contributions of different mechanisms change: the iso-contours of m^{loop} do not move, the see-saw lines shift to smaller M as $M \propto m^2$, whereas the iso-contours of tree level contribution shift as $M \propto m$, *i.e.* weaker. Therefore, the tree-level contribution becomes important and can dominate for a small Dirac mass m in the range of a small RH Majorana mass M and relatively large ξ. With increase of m_4, $\xi^{\text{loop}} \propto 1/m_4^2$ decreases faster than $\xi^{\text{tree}} \propto 1/m_4$. Therefore the tree level contribution becomes substantial and the allowed region shifts to smaller ξ. Also with increase of the charged lepton mass m_E the loop contribution increases.

In Fig. 4 we show the iso-contours of different contributions to light neutrino masses in the $m_4 - M$ plane for fixed $m = 31\,\mathrm{MeV}$ and $m_E = m_4 + 200\,\mathrm{GeV}$. As in the Fig. 3, the contours of the tree level contribution can be obtained from Eq. (40) and the contours



(a) $\xi = 10^{-7}$ (b) $\xi = 10^{-9}$

Figure 4: Iso-contours of the contributions to light neutrino masses (numbers at the curves) from different mechanisms in the $m_4 - M$ plane for two different values of the mixing parameter ξ. The Dirac mass m is fixed to $m = 31\,\mathrm{MeV}$ and the fourth generation charged lepton mass m_E is fixed to be 200 GeV larger than m_4: $m_E = m_4 + 200$ GeV. Black dashed lines correspond to the tree-level contribution, blue solid lines to the loop contribution and red dotted ones to the usual see-saw contribution. Note that the Higgs mass m_H has been neglected in the calculation of the radiatively induced M_4. Hence for $M \sim m_H$, the loop contribution can only be considered as an order of magnitude estimate.

of the loop contribution can be read off from Eq. (41). The usual see-saw contribution, $m^{\mathrm{ss}} = -m^2/M$, does not depend on m_4 and the corresponding iso-contours are just vertical lines. The iso-contours are coloured in the same way as in Fig. 3.

Since the loop effect alone cannot explain neutrino data, the other contributions (the see-saw or/and tree level) should be present and without strong suppression.

For large RH neutrino masses the loop contribution to the light neutrino masses dominates over the usual see-saw contribution as well as the tree-level contribution. In particular, the two loop contribution is incompatible with RH neutrino masses close to the GUT scale, unless the fourth generation effectively decouples, *i.e.* $\xi \lesssim 10^{-12}$. Hence, the combination of LH and RH mixing angles of the fourth generation with the three light SM generations is highly constrained by this contribution. The tree-level contribution of the fourth generation is only relevant for small RH neutrino masses (particularly below 100 TeV for the values

fixed in Fig. 3) and therefore small Dirac masses.

Note that contributions of the fourth generation to the light neutrino masses (both tree level and loops if $M_4 = M_4^{\rm loop}$, see Eq. (12) and Eq. (26)) are proportional to mixing of the RH neutrinos. Therefore in the limit $U_R \to 0$, the light neutrino masses are generated by the usual see-saw mechanism and the left mixing of the fourth generation with the first three generations can be large: at the level of the upper bounds.

3.4 3 + 1 Generation Case: an Example

The results "1+1" generations presented in the previous section can be also used in the analysis of the (3 + 1) case. Here we present an example, where the two loop contribution of the fourth generation is essential for neutrino masses.

We assume that three massive RH neutrinos have the common Majorana mass $M_0 = 10^9$GeV and the Majorana mass of the fourth RH neutrino is zero at tree-level. In the flavour basis, the neutrino Dirac mass matrix of the three light SM generations is given by the democratic mass matrix with the common mass scale of $m = 31\,\text{MeV}$, so that the usual see-saw contribution equals

$$m^{\rm ss} = -0.00288 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \text{eV} \,. \tag{45}$$

We take the Dirac mass of the fourth generation to be $m_4 = 400\,\text{GeV}$ and the fourth generation charged lepton mass, $m_E = 600\,\text{GeV}$. For the RH mixing of the fourth generation $(U_R)_{\alpha 4} \simeq 1.08{\cdot}10^{-4}$ and the left-handed mixing $(U_L) \simeq 1.08{\cdot}10^{-4}{\cdot}(0.15,\ 1,\ -1)$, the two-loop contribution with a cutoff scale $\Lambda = 10\,M_0$ equals

$$m^{\rm loop} = -0.02473 \begin{pmatrix} 0.0225 & 0.15 & -0.15 \\ 0.15 & 1 & -1 \\ -0.15 & -1 & 1 \end{pmatrix} \text{eV} \,. \tag{46}$$

The tree-level contribution of the fourth generation is of the order of 10^{-6}eV, and therefore, negligible.

The loop and see-saw contributions have rank 1 and their combination leads to a strong normal mass hierarchy with the mass splittings $\Delta m^2_{31} = 2.50 \cdot 10^{-3}$ eV^2 and $\Delta m^2_{21} = 7.41 \cdot 10^{-5}$ eV^2, and mixing angles $\sin^2\theta_{12} = 0.330$, $\sin^2\theta_{13} = 0.013$, and $\sin^2\theta_{23} = 0.510$ in agreement with observations.

4 The Fourth Generation and Symmetries

In spite of many efforts to explain the observed features of lepton mixing using various discrete flavour symmetries, no convincing model has been proposed so far (see [65] for recent review). In this connection, we will explore whether the existence of 4th generation can help in the realisation of discrete flavour symmetries. Existence of four generations of fermions can be explained if the flavour symmetry group has the lowest irreducible representation $\underline{\mathbf{4}}$ (apart from singlet representations). The key feature here is very small mass of one right handed neutrino, $M_4 \ll M_k$, which can be a consequence of a certain symmetry. In this connection, a natural question is whether the same symmetry which leads to $M_4 \ll M_k$ can produce certain flavour structures for the three light generations?

In the following, we will present some general results on model-building in the four generation context and then focus on the simplest symmetry group in detail.

4.1 General Comments

As we have shown in Sec. 3, the following features are important for model building:

- The RH neutrino mass matrix should be nearly singular. Three massive and one (almost) massless RH neutrino are required.
- The two loop contribution to the light neutrino mass matrix has rank 1 and its flavour structure is given by f_L, that is, by the LH mixing of the 4th generation with the three light SM generations.
- The tree-level contribution to m_ν produced by mixing of light neutrinos with fourth generation neutrino is negligible compared to the two loop contribution for large RH neutrino masses, and it becomes important only for small M_k, as it can be seen in Fig. 3 and Fig. 4.

The simplest possibility to obtain a pseudo-Dirac structure for the fourth neutrino is to impose the conservation of the fourth generation lepton number, L_4. This implies decoupling of the 4th generation. Breaking of the lepton number symmetry is then needed to mix the 4th neutrino with the light neutrinos. This leads to the tree level contribution m^{tree}, and possibly to the generation of a 4th generation Majorana mass term, which in turn produces a two loop contribution. The spontaneous breaking of the global $\mathrm{U}(1)_{L_4}$ symmetry results in a Goldstone boson (Majoron). This Majoron is not dangerous, because it couples directly to the fourth generation only and its coupling with the three light SM generations is suppressed:

$$g_{\alpha\beta} \sim U_{\alpha 4} U_{\beta 4} M_4 / m_4 .$$

Notice that this coupling has a similar dependence on $U_{\alpha 4}U_{\beta 4}M_4$ as the 2-loop contribution. For $M_4 = 1$ GeV and $(U_L)^2_{\alpha 4} \sim 10^{-7}$, one obtains $g_{\alpha\beta} \lesssim 10^{-8}$ which satisfies limits on the Majoron couplings [66]. Different values of M_4 lead to a similar limit on the coupling $g_{\alpha\beta}$ due to the similar dependence on $U_{\alpha 4}U_{\beta 4}M_4$. The $\mathrm{U}(1)_{L_4}$ symmetry cannot be gauged, unless additional particles are introduced to cancel the anomalies.

An abelian symmetry can only forbid certain terms in the mass matrix and produce a mass hierarchy but cannot lead to relations between different elements of the matrix. In this connection, we consider non-abelian groups with the lowest non-trivial irreducible representation $\underline{\mathbf{4}}$. This (i) explains existence of four generations, and (ii) opens up a possibility to obtain certain flavour structures.

We use the SmallGroups catalogue of `GAP` [67] to obtain the groups with irreducible representation $\underline{\mathbf{4}}$ in a systematic way. They are denoted as $\mathrm{SG}(N, m)$, where N is the order and m is the index of the group in the SmallGroups catalogue. We find that the smallest group with a 4 dimensional representation is $\mathrm{SG}(20, 3) \cong \mathbb{Z}_5 \rtimes_\varphi \mathbb{Z}_4$. It does not contain other non-singlet representations besides $\underline{\mathbf{4}}$ and has order 20. Hence, it is much smaller than A_5, which has been studied in [51]. The next groups with a real four-dimensional representation are of order 32:

$$\begin{aligned}
\mathrm{SG}(32,6) &\cong ((\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes_\varphi \mathbb{Z}_2) \rtimes_\varphi \mathbb{Z}_2 &\qquad \mathrm{SG}(32,44) &\cong (\mathbb{Z}_2 \times Q_8) \rtimes_\varphi \mathbb{Z}_2 \\
\mathrm{SG}(32,7) &\cong (\mathbb{Z}_8 \rtimes_\psi \mathbb{Z}_2) \rtimes_\varphi \mathbb{Z}_2 & \mathrm{SG}(32,49) &\cong (\mathbb{Z}_2 \times D_4) \rtimes_\varphi \mathbb{Z}_2 \\
\mathrm{SG}(32,8) &\cong (\mathbb{Z}_2 \times \mathbb{Z}_2) \rtimes_\varphi (\mathbb{Z}_4 \times \mathbb{Z}_2) & \mathrm{SG}(32,50) &\cong (\mathbb{Z}_2 \times Q_8) \rtimes_\varphi \mathbb{Z}_2 \\
\mathrm{SG}(32,43) &\cong (\mathbb{Z}_2 \times D_4) \rtimes_\varphi \mathbb{Z}_2 \,, & &
\end{aligned} \tag{47}$$

where the defining homomorphism φ/ψ of each semi-direct product is not specified explicitly. The smallest groups with a complex four-dimensional representation are of order 60:

$$\mathrm{SG}(60,6) \cong \mathbb{Z}_3 \times (\mathbb{Z}_5 \rtimes_\varphi \mathbb{Z}_4) \qquad \mathrm{SG}(60,7) \cong \mathbb{Z}_{15} \rtimes_\varphi \mathbb{Z}_4 \qquad \mathrm{SG}(60,8) \cong S_3 \times D_5 \,. \tag{48}$$

In the following, we will concentrate on the smallest group $\mathrm{SG}(20, 3)$.

4.2 The Smallest Group: $\mathrm{SG}(20, 3) \cong \mathbb{Z}_5 \rtimes_\varphi \mathbb{Z}_4$

The smallest group with a four-dimensional representation, $\mathrm{SG}(20, 3)$, is the Frobenius group of order 20, with presentation

$$\left\langle s, t | s^4 = t^5 = \mathbb{1}, ts = st^2 \right\rangle, \tag{49}$$

which can be considered as a subgroup of S_5 generated by

$$\langle (2, 3, 5, 4), (1, 2, 3, 4, 5) \rangle \ . \tag{50}$$

The decomposition of the Kronecker product $\underline{\mathbf{1_i}} \times \underline{\mathbf{1_j}}$ equals

$$\left(\underline{\mathbf{1_1}} \;\; \underline{\mathbf{1_2}} \;\; \underline{\mathbf{1_3}} \;\; \underline{\mathbf{1_4}} \right) \times \begin{pmatrix} \underline{\mathbf{1_1}} \\ \underline{\mathbf{1_2}} \\ \underline{\mathbf{1_3}} \\ \underline{\mathbf{1_4}} \end{pmatrix} = \begin{pmatrix} \underline{\mathbf{1_1}} & \underline{\mathbf{1_2}} & \underline{\mathbf{1_3}} & \underline{\mathbf{1_4}} \\ \cdots & \underline{\mathbf{1_1}} & \underline{\mathbf{1_4}} & \underline{\mathbf{1_3}} \\ \cdots & \cdots & \underline{\mathbf{1_2}} & \underline{\mathbf{1_1}} \\ \cdots & \cdots & \cdots & \underline{\mathbf{1_2}} \end{pmatrix} , \tag{51}$$

the Kronecker product of $\underline{\mathbf{4}}$ with any of the singlet representations is given by

$$\underline{\mathbf{4}} \times \underline{\mathbf{1_i}} = \underline{\mathbf{4}} \,, \tag{52}$$

and the non-trivial Kronecker product of $\underline{\mathbf{4}} \times \underline{\mathbf{4}}$ is

$$\{\underline{\mathbf{4}} \times \underline{\mathbf{4}}\} = \underline{\mathbf{1_1}} \oplus \underline{\mathbf{1_2}} \oplus \underline{\mathbf{4}}_S \oplus \underline{\mathbf{4}}_S \qquad\qquad [\underline{\mathbf{4}} \times \underline{\mathbf{4}}] = \underline{\mathbf{1_3}} \oplus \underline{\mathbf{1_4}} \oplus \underline{\mathbf{4}}_A, \tag{53}$$

where { } denotes symmetrisation and [ ] – antisymmetrisation. The other group theoretical details of SG(20, 3) are summarised in App. A.

Note that SG(20, 3) does not contain any subgroup with an irreducible representation $\underline{\mathbf{3}}$ because 20 is not divisible by 3. Therefore it cannot be broken down to $\underline{\mathbf{3}} + \underline{\mathbf{1}}$, and consequently specific properties of the 4th generation compared to the three other generations cannot be explained as immediate consequence of the symmetry breaking.

4.3 Flavour Structures and SG(20, 3)

Let us find possible flavour structures (structures of the fermion mass matrices) which can be obtained with SG(20, 3) symmetry. The required Clebsch-Gordan coefficients are given in App. A.

1) In the limit of exact symmetry, the operators which lead to fermion masses have the form mF_1F_2. (We omit the usual non-flavoured Higgs fields, which should be added to satisfy gauge invariance.) Here F_1 and F_2 are fermion multiplets transforming under a certain representation of SG(20, 3). If F_i form quartets, $F_i \sim \underline{\mathbf{4}}$, the mass operators has group structure $\underline{\mathbf{4}} \times \underline{\mathbf{4}}$ and leads to the symmetric non-singular mass matrix:

$$\begin{pmatrix} 0 & 0 & m & 0 \\ 0 & 0 & 0 & m \\ m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \end{pmatrix} . \tag{54}$$

Apparently it cannot be used for RH neutrinos. If F_i are singlets of SG(20, 3), $F_i \sim \underline{\mathbf{1_1}} \oplus \underline{\mathbf{1_2}} \oplus \underline{\mathbf{1_3}} \oplus \underline{\mathbf{1_4}}$ the mass operators, $(\underline{\mathbf{1_1}} \oplus \underline{\mathbf{1_2}} \oplus \underline{\mathbf{1_3}} \oplus \underline{\mathbf{1_4}}) \times (\underline{\mathbf{1_1}} \oplus \underline{\mathbf{1_2}} \oplus \underline{\mathbf{1_3}} \oplus \underline{\mathbf{1_4}})$, generate the mass matrix

$$\begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & m_4 \\ 0 & 0 & m_3 & 0 \end{pmatrix} . \tag{55}$$

If one of the mass parameters vanishes, $m_i = 0$, this matrix has a vanishing eigenvalue. It can be used to describe three massive and one massless Majorana RH neutrinos. In the case $F_1 = F_2 = N$, the matrix (55) is symmetric and the condition $m_1 = 0$ or $m_2 = 0$ should be satisfied.

2) Let us consider operators of the type $yF_1F_2\chi$ with flavon fields χ which transform non-trivially under SG(20, 3). They generate the mass terms, when the flavour symmetry is broken: $\langle\chi\rangle \neq 0$. We introduce the quartet flavons, $\phi \sim \underline{\mathbf{4}}$, and the singlets $\chi_i \sim \underline{\mathbf{1}}_{\mathbf{i}}$, with $i = 1, 2, 3, 4$, and denote the VEVs of these fields as $\langle\chi_i\rangle = \langle\underline{\mathbf{1}}_{\mathbf{i}}\rangle = u_i$ and $\langle\phi\rangle = \langle\underline{\mathbf{4}}\rangle = \begin{pmatrix} v_1, & v_2, & v_3, & v_4 \end{pmatrix}$. If $F_1, F_2 \sim \underline{\mathbf{4}}$, then the following invariant operators can be introduced:

$$2\, y_j(F_1F_2)_j\chi_j \quad j = 2, 3, 4, \quad y_{S1}\{F_1F_2\}\phi, \quad \sqrt{2}y_{S2}\{F_1F_2\}\phi, \quad \sqrt{2}y_A[F_1F_2]\phi, \tag{56}$$

where y_i are the Yukawa couplings, and in the second operator there are three different possibilities of pairing. (The operator with $j = 1$ gives the structure (55) without symmetry breaking.) The operators (56) produce the matrix

$$\begin{pmatrix} v_2y_{S1} & v_1(y_{S2}+y_A) & u_2y_1+u_3y_3+u_4y_4 & v_4(y_{S2}-y_A) \\ v_1(y_{S2}-y_A) & v_3y_{S1} & v_2(y_{S2}+y_A) & -u_2y_2-\mathrm{i}u_3y_3+\mathrm{i}u_4y_4 \\ u_2y_2-u_3y_3-u_4y_4 & v_2(y_{S2}-y_A) & v_4y_{S1} & v_3(y_{S2}+y_A) \\ v_4(y_{S2}+y_A) & \mathrm{i}\,(u_3y_3-u_4y_4)-u_2y_2 & v_3(y_{S2}-y_A) & v_1y_{S1} \end{pmatrix} \tag{57}$$

If $F_1, F_2 \sim \oplus_{i=1}^4 \underline{\mathbf{1}}_{\mathbf{i}}$ are 4 different singlets of the symmetry group, then the symmetry structure of the fermionic part of the operator is $\oplus_{i=1}^4 \underline{\mathbf{1}}_{\mathbf{i}} \times \oplus_{i=1}^4 \underline{\mathbf{1}}_{\mathbf{i}}$ and only singlet flavon fields can be used. The invariant combinations

$$\begin{aligned} &y_1(F_1)_2(F_2)_1\chi_2, \quad y_2(F_1)_1(F_2)_2\chi_2, \quad y_3(F_1)_3(F_2)_3\chi_2, \quad y_4(F_1)_4(F_2)_4\chi_2, \\ &y_5(F_1)_4(F_2)_1\chi_3, \quad y_6(F_1)_3(F_2)_2\chi_3, \quad y_7(F_1)_2(F_2)_3\chi_3, \quad y_8(F_1)_1(F_2)_4\chi_3, \\ &y_9(F_1)_3(F_2)_1\chi_4, \quad y_{10}(F_1)_4(F_2)_2\chi_4, \quad y_{11}(F_1)_1(F_2)_3\chi_4, \quad y_{12}(F_1)_2(F_2)_4\chi_4 \end{aligned} \tag{58}$$

generate the mass matrix

$$\begin{pmatrix} 0 & u_2y_2 & u_4y_{11} & u_3y_8 \\ u_2y_1 & 0 & u_3y_7 & u_4y_{12} \\ u_4y_9 & u_3y_6 & u_2y_3 & 0 \\ u_3y_5 & u_4y_{10} & 0 & u_2y_4 \end{pmatrix}. \tag{59}$$

The value of each matrix element is independent; and if only one flavon field χ_i is introduced (i.e only one u_i in the matrix above is non-zero) the symmetry only demands that four matrix elements are generated. Apparently there is no contribution from the flavon ϕ. Finally, if $F_1 \sim \oplus_{i=1}^4 \underline{\mathbf{1}}_{\mathbf{i}}$ and $F_2 \sim \underline{\mathbf{4}}$, the fermionic flavour structure $\underline{\mathbf{4}} \times \oplus_{i=1}^4 \underline{\mathbf{1}}_{\mathbf{i}}$ requires the quartet of flavons ϕ. The operators

$$y_1\{F_2\phi\}_1(F_1)_1, \quad y_2\{F_2\phi\}_2(F_1)_2, \quad y_3[F_2\phi]_4(F_1)_3, \quad y_4[F_2\phi]_3(F_1)_4 \tag{60}$$

produce the mass matrix

$$\frac{1}{2}\begin{pmatrix} v_3 y_1 & v_4 y_1 & v_1 y_1 & v_2 y_1 \\ v_3 y_2 & -v_4 y_2 & v_1 y_2 & -v_2 y_2 \\ v_3 y_3 & -i v_4 y_3 & -v_1 y_3 & i v_2 y_3 \\ v_3 y_4 & i v_4 y_4 & -v_1 y_4 & -i v_2 y_4 \end{pmatrix} . \tag{61}$$

Several important conclusions can be drawn from the forms of these mass matrices. A singular Majorana mass matrix with only one vanishing mass eigenvalue cannot be obtained as an immediate result of the $\mathrm{SG}(20,3)$ symmetry or its breaking without additional assumptions or symmetries. Such a matrix can be obtained by tuning of couplings which in turn requires introduction of additional symmetries. For example, suppose the RH neutrinos transform as $\underline{\mathbf{4}}$ and the direct mass terms are somehow forbidden so that the leading order contribution comes from the one flavon insertion $\phi \sim \underline{\mathbf{4}}$. Then according to Eq. (57) , one massless RH neutrino can be obtained with the VEV alignment $\langle\phi\rangle = v\,(1,\,1,\,1,\,1)$, and equality $y_{S1} = \pm y_{S2}$ of the Yukawa couplings of the two possible (symmetric $\{NN\}$) invariants. A study of the simplest potential with one four-dimensional representation shows that the only allowed VEV configuration is indeed $\langle\phi\rangle = v\,(1,\,1,\,1,\,1)$, which breaks $\mathrm{SG}(20,3)$ to $\mathbb{Z}_4$, as it is shown in Tab. 1, unless there are special relations between parameters in the flavon potential.

4.4 Models and Phenomenology

In the following, we discuss the leading order predictions for different symmetry assignments for the fields. Let us assign for leptons the following transformation properties: $\ell \sim \underline{\mathbf{4}}$ and $e_R \sim \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{3}} + \underline{\mathbf{1}}_{\mathbf{4}} + \underline{\mathbf{1}}_{\mathbf{1}}$, which allows to generate different charged lepton masses and explain the number of generation. We use flavons $\phi \sim \underline{\mathbf{4}}$ and $\chi_i \sim \underline{\mathbf{1}}_{\mathbf{i}}$.

1) If the RH neutrinos transform as singlets $N \sim \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{3}} + \underline{\mathbf{1}}_{\mathbf{4}} + \underline{\mathbf{1}}_{\mathbf{1}}$, the RH Majorana mass matrix has full rank. In order to obtain a singular RH neutrino mass matrix, we set M_4 to zero (which according to our symmetry assignment corresponds to parameter m_1 in matrix (55)). This can be obtained in different ways: (i) by choice, (ii) by a "missing" representation, *i.e.* by choosing the assignment of representations of N in such a way that there is only one of the complex conjugate representations $\underline{\mathbf{1}}_{\mathbf{3,4}}$ or (iii) by an additional symmetry, *e.g.* $N_4 \to \mathrm{i}\,N_4$, which we are going to discuss in the following. This effectively leads to a $3+1$ structure of the RH neutrinos and forbids several couplings in the Dirac mass matrices. Therefore, we demand the following transformation properties $\phi \to -\mathrm{i}\phi$, $E_4 \to \mathrm{i}E_4$ as well as introduce another flavon $\eta \sim \underline{\mathbf{1}}_{\mathbf{1}}$ transforming as $\eta \to \mathrm{i}\eta$. All other fields are invariant under the additional symmetry. The leading order Lagrangian is given

by

$$-\mathcal{L} = M_1 N_1^T N_1 + M_2\left(N_2^T N_3 + N_3^T N_2\right) + Y_4^\nu \bar{\ell} N_4 H_1 \frac{\phi}{\Lambda} + Y_4^l \bar{\ell} H_2 e_{4R}\frac{\phi}{\Lambda} + Y_k^\nu \bar{\ell} N_k H_1 \frac{\phi\eta}{\Lambda^2} + Y_k^l \bar{\ell} H_2 e_{kR}\frac{\phi\eta}{\Lambda^2} + \text{h.c.} \quad (62)$$

with $k = 1, 2, 3$. Here Y_j^l (j = 1, 2, 3, 4) are the charged lepton Yukawa couplings and Y_i^ν (i = 1, 2, 3, 4) are the neutrino Yukawa couplings. This leads to the following 4×4 mass matrices in the basis $\nu_{e,\mu,\tau,E}$ and $N_{1,2,3,4}$ for $\langle\phi\rangle = v\,(1,\,1,\,1,\,1)$ and $\langle\eta\rangle = u$:

$$M = \begin{pmatrix} M_1 & 0 & 0 & 0 \\ 0 & 0 & M_2 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad m = \frac{v\, v_{\text{EW}}^\nu}{2\Lambda}\begin{pmatrix} Y_1\frac{u}{\Lambda} & Y_2\frac{u}{\Lambda} & Y_3\frac{u}{\Lambda} & Y_4 \\ -Y_1\frac{u}{\Lambda} & -iY_2\frac{u}{\Lambda} & iY_3\frac{u}{\Lambda} & Y_4 \\ Y_1\frac{u}{\Lambda} & -Y_2\frac{u}{\Lambda} & -Y_3\frac{u}{\Lambda} & Y_4 \\ -Y_1\frac{u}{\Lambda} & iY_2\frac{u}{\Lambda} & -iY_3\frac{u}{\Lambda} & Y_4 \end{pmatrix}. \quad (63)$$

The charged lepton mass matrix has the same structure as the Dirac neutrino mass matrix.

In the basis, where the mass matrices of charged leptons and the RH neutrinos are diagonal: $m_e^{fl} = \frac{v\, v_{\text{EW}}^e}{2\Lambda}\,\text{diag}(Y_1^l u/\Lambda,\, Y_2^l u/\Lambda,\, Y_3^l u/\Lambda,\, Y_4^l)$, $M^{fl} = \text{diag}(M_1,\, M_2,\, M_2,\, 0)$, the Dirac neutrino mass matrix becomes

$$m^{fl} = \frac{v\, v_{\text{EW}}^\nu}{\Lambda}\begin{pmatrix} Y_1^\nu\frac{u}{\Lambda} & 0 & 0 & 0 \\ 0 & -\frac{iY_2^\nu u}{\sqrt{2}\Lambda} & \frac{Y_2^\nu u}{\sqrt{2}\Lambda} & 0 \\ 0 & \frac{iY_3^\nu u}{\sqrt{2}\Lambda} & \frac{Y_3^\nu u}{\sqrt{2}\Lambda} & 0 \\ 0 & 0 & 0 & Y_4^\nu \end{pmatrix}, \quad (64)$$

where $v_{\text{EW}}^\nu \equiv \langle H_1\rangle$ and $v_{\text{EW}}^e \equiv \langle H_2\rangle$. It is not diagonal like the charged lepton mass matrix due to the additional rotation from diagonalising the RH neutrino mass matrix. Its structure corresponds to $f_L = f_R = 0$ in our general consideration of Sec. 2. Hence, the fourth generation decouples from the first three generations and the only contribution to the light neutrino mass matrix originates from the ordinary see-saw mechanism

$$m^{\text{ss}} = \frac{{v_{\text{EW}}^\nu}^2 v}{\Lambda^2}\begin{pmatrix} A & 0 & 0 \\ \dots & 0 & B \\ \dots & \dots & 0 \end{pmatrix}. \quad (65)$$

The coefficients A and B are given by $A = -Y_1^2\frac{u^2 v}{M_1\Lambda^2}$ and $B = -Y_2 Y_3\frac{u^2 v}{M_3\Lambda^2}$, respectively, and lead to a vanishing atmospheric mass squared difference.

This mass squared difference can be obtained by the introduction of flavon $\chi_2 \sim \underline{\mathbf{1}}_{\mathbf{2}}$, whose different couplings $N_2^T N_2 \chi_2$ and $N_3^T N_3 \chi_2$ split masses of the RH neutrinos N_2 and N_3 which otherwise equal M_2 (see M^{fl} above). This split in turn generates non-zero elements $m_{\mu\mu}^{\text{ss}}$ and $m_{\tau\tau}^{\text{ss}}$ in the see-saw matrix (65). If flavon singlets $\chi_k \sim \underline{\mathbf{1}}_{\mathbf{k}}$ with $k = 2, 3, 4$ and $\langle\chi_k\rangle = u_k$ are introduced, which transform under additional symmetry as $\chi_k \to -\text{i}\chi_k$,

mixing between the fourth generation and the three light generations is generated in the RH Majorana mass matrix by interactions $N_4^T(h_2\chi_2 N_1 + h_4\chi_4 N_2 + h_3\chi_3 N_3)$ with Yukawa couplings h_k. Also these additional flavons contribute to the 3×3 block of the first three generations in the Dirac mass matrices because $\phi\chi_k^\dagger$ is invariant under the additional symmetry. This leads to appearance of $f_{L,R}$ as well as M_4 due to the mixing of N_4 with N_k. Hence, there are the rank 1 two loop contribution and the tree level contribution to neutrino masses (see in Sec. 2). As the tree level contribution is generated at a higher order in flavon insertions compared to the see-saw contribution, it can be neglected at leading order.

Let us comment on other possible VEV alignments and structure of the neutrino Dirac mass matrix. Any VEV alignment of the quartet, which differs from equality of components, induces mixing between the fourth neutrino and the three light ones, which is proportional to the deviation from the VEV alignment $\langle\phi\rangle = v\,(1,\,1,\,1,\,1)$ besides generating the elements $m^{\rm ss}_{\mu\mu}$ and $m^{\rm ss}_{\tau\tau}$. However, the constraints on the mixing between the fourth and the three light SM generations does not allow large enough values for $m^{\rm ss}_{\mu\mu}$ and $m^{\rm ss}_{\tau\tau}$ without introducing additional flavons $\chi_k \sim \underline{\mathbf{1}}_{\mathbf{k}}$.

Summarising, the simplest construction with only one flavon $\phi\sim\underline{\mathbf{4}}$ does give correct values of neutrino masses and mixing.

2) On the contrary, suppose the RH neutrinos transform as $N\sim\underline{\mathbf{4}}$ and the direct mass term NN is forbidden by an additional auxiliary symmetry, *e.g.* $N\to\omega N$ with $\omega = e^{2\pi \mathrm{i}/3}$ in order to achieve a singular RH neutrino mass matrix, the Lagrangian is given by

$$-\mathcal{L} = \frac{1}{2}h_1\{N^T N\}_{\rm diag}\phi + h_2\{N^T N\}_{\rm off-diag}\phi + Y^\nu_k(\bar\ell N)_k H_1\frac{\phi^\dagger}{\Lambda} + Y^l_i\bar\ell H_2 e_{iR}\frac{\phi^\dagger}{\Lambda} + \text{h.c.}\,, \tag{66}$$

where $\phi\sim\underline{\mathbf{4}}$ and $\phi\to\omega\phi$ as well as $e_{iR}\to\omega e_{iR}$. Y^ν_k $(k = S1, S2, A)$ correspond to three possible combinations of $(\bar\ell N)$ which transform as $\underline{\mathbf{4}}_S$, $\underline{\mathbf{4}}_S$, $\underline{\mathbf{4}}_A$ (see Eq. (53)). Taking the VEV alignment $\langle\phi\rangle = v\,(1,\,1,\,1,\,1)$, the RH neutrino mass matrix becomes

$$M = \frac{v}{4}\begin{pmatrix} h_1 & h_2 & 0 & h_2 \\ \dots & h_1 & h_2 & 0 \\ \dots & \dots & h_1 & h_2 \\ \dots & \dots & \dots & h_1 \end{pmatrix} \tag{67}$$

with mass eigenvalues $|h_1|v$, $|h_1|v$, $|h_1-2h_2|v$ and $|h_1+2h_2|v$. Hence, there is exactly one massless RH neutrino if $h_1=\pm 2h_2$. Under the assumption $h_1=-2h_2$, which can be obtained by fine-tuning the couplings $h_{1,2}$ only, the diagonalised RH neutrino mass matrix becomes $M^{fl} = |h_1|v\,\mathrm{diag}(2,1,1,0)$. The eigenstate corresponding to the zero mass eigenvalue is $(1,\,1,\,1,\,1)$. In flavour basis, where the charged leptons and the RH neutrino mass matrix are diagonal, the charged lepton mass matrix is $m_e^{fl} = \frac{v\,v^e_{\rm EW}}{2\Lambda}\,\mathrm{diag}(Y^e_1,\,Y^e_2,\,Y^e_3,\,Y^e_4)$

and the Dirac neutrino mass matrix is given by

$$m^{fl} = \frac{v\, v_{\rm EW}^\nu}{2\Lambda} \begin{pmatrix} 2Y_{S1}^\nu - Y_{S2}^\nu & 0 & 0 & 0 \\ 0 & -\frac{2Y_A^\nu + iY_{S2}^\nu}{\sqrt{2}} & \frac{2iY_A^\nu - Y_{S2}^\nu}{\sqrt{2}} & 0 \\ 0 & \frac{-2Y_A^\nu + iY_{S2}^\nu}{\sqrt{2}} & \frac{-2iY_A^\nu - Y_{S2}^\nu}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 2Y_{S1}^\nu + Y_{S2}^\nu \end{pmatrix} . \tag{68}$$

This leads to a see-saw contribution to the neutrino mass matrix of the form (65) with coefficients $A = -\frac{(Y_{S2}-2Y_{S1})^2}{2h_1}$ and $B = -\frac{4Y_A^2+Y_{S2}^2}{h_1}$. Hence, it has the same structure with different coefficients and, essentially, the same conclusions can be drawn as in case 1). Similarly to the previous assignment of representations, the addition of flavons $\chi_k \sim \underline{\mathbf{1}}_{\mathbf{k}}$ will lead to a non-vanishing atmospheric mass squared difference and a mixing of the fourth generation with the first three SM generations.

Concluding, the simplest group SG$(20,3)$ does not immediately lead to the required flavour structure. For instance tuning of Yukawa couplings is required to obtain zero mass for one of the RH neutrinos if they transform as $\underline{\mathbf{4}}$. In this case, the direct mass term of a four dimensional representation always has full rank, and the RH neutrino mass matrix has to be generated using non-singlet flavons. Furthermore, an additional leading order contribution to the neutrino mass matrix is required which generates the $(m_\nu)_{\mu\mu}$ and $(m_\nu)_{\tau\tau}$ entries. We have demonstrated how those contributions can be achieved.

A survey of all small groups up to order 56 with the RH neutrinos transforming as $\underline{\mathbf{4}}$ shows that the RH neutrino masses are either all of the same order of magnitude or there are two heavy and two light RH neutrinos. Hence, one might argue that $\underline{\mathbf{4}}$ is generally not the best representation for the RH neutrinos and a more viable choice is $\underline{\mathbf{3}} \oplus \underline{\mathbf{1}}'$, where $\underline{\mathbf{3}}$ is a real representation of a given group and $\underline{\mathbf{1}}'$ a complex representation such that the matrix of direct mass terms is singular with one vanishing mass. This splitting might also be obtained from breaking the symmetry group to a smaller subgroup. The smallest group, which allows the decomposition of $\underline{\mathbf{4}}$ into $\underline{\mathbf{3}} \oplus \underline{\mathbf{1}}$, is A_5. However A_5 has irreducible representations $\underline{\mathbf{3}}$ as well as $\underline{\mathbf{5}}$, so that use of representation $\underline{\mathbf{4}}$ only should be justified.

5 Summary and Conclusions

1. We have explored the generation of light neutrino masses in the presence of a fourth family of fermions with four RH neutrinos (1 per family). In this context, generically there are three contributions to the light neutrino masses from three different mechanisms:

(i) the usual high mass scale see-saw contribution;

(ii) the tree level contribution induced by mixing of the light generations with the fourth generation. This contribution requires mixing of both left and right neutrino com-

ponents ($f_L \neq 0$ $f_R \neq 0$) in the basis where the Majorana mass matrix of the RH components is diagonal.

(iii) The two loop contribution with two W-bosons exchange, related to the non-zero Majorana mass of the fourth neutrino, M_4.

2. We show that even if $M_4 = 0$ initially at tree level, it will be generated at the two loop level due to usual Yukawa interactions. This radiatively generated mass is proportional to the large Majorana masses M_i and therefore, is rather large: $0.1 - 1$ GeV. Unless there is strong cancellation, (*e.g.*, with tree level contribution), this mass, in turn, produces the dominant contribution to the light neutrino masses in large part of parameter space. In the case $f_R = 0$, the new contributions related to the fourth generation vanish. The relative contributions from different mechanisms depend strongly on $M, m, \xi \equiv (U_L)_{\alpha 4}(U_R)_{i4}$, and to a smaller degree on m_E and m_4. The tree level contribution of the fourth generation dominates over the loop contribution for small RH neutrino masses M and vice versa, as it is illustrated in Figs. 3 and 4. The smaller Dirac mass, m, the larger the tree level contribution compared to the usual see-saw contribution; the loop contribution is independent of m. The usual see-saw contribution does not depend on ξ, while the tree level contribution and the loop contribution are proportional to ξ and ξ^2 respectively.

3. In general, the contributions from three different mechanisms have different flavour structures. The loop contribution is singular and therefore it cannot explain the observed mass hierarchy. Therefore, comparable contributions should follow from other mechanisms. The combination of the loop and see-saw contributions is realized at $M_i \sim (10^7 - 10^{10})$ GeV, a large Dirac neutrino mass m, and $\xi = 10^{-9} - 10^{-7}$. Combination of the loop and tree level contributions works for $M_i \lesssim 10^5$ GeV (and therefore small m) and $\xi = 10^{-6}$. An interplay of the "see-saw and tree level" contributions is realized at small Dirac neutrino mass m and $\xi < 10^{-7}$. The loop contribution gives very strong bound on mixing parameters especially for large M_i. At $M_i > 10^4$ GeV the bounds are much stronger than those from the direct searches. The loop contribution can be suppressed if certain cancellation occurs between contributions from different M_i or between the loop and tree level contribution, although this looks rather unnatural.

The upper bound on the light neutrino masses (which follows from cosmology) gives the most stringent bound on the parameters of the model.

4. In the see-saw limit of the fourth generation, where the Majorana mass M_4 is much larger than the Dirac mass m_4, $M_4 \gg m_4$, the tree-level contribution of the fourth generation to the three light active neutrinos is negligible. In the pseudo-Dirac limit, $m_4 \gg M_4$, the tree-level contribution of the 4th generation to the light neutrino mass matrix is significant for small RH neutrino masses M_i and determines one of the mass scales.

5. We explored flavour symmetries, which could explain the leptonic flavour structure and studied the smallest group with a four-dimensional representation, $\mathrm{SG}(20, 3) \cong \mathbb{Z}_5 \rtimes_\varphi \mathbb{Z}_4$. We have found the simplest flavour structures (mass matrices), which can be obtained as a

result of this flavour symmetry. The required singularity of the RH neutrino mass matrix can be achieved imposing conditions on the Yukawa couplings and VEVs. There is no viable model based on one flavon $\phi \sim \underline{\mathbf{4}}$, but there are phenomenologically viable models can be constructed with flavons $\phi \sim \underline{\mathbf{4}}$ and $\chi_k \sim \underline{\mathbf{1}}_{\mathbf{k}}$. We indicated the next smallest groups, which might be interesting to study.

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A Group Theory of $\mathbf{SG(20,3)} \cong \mathbb{Z}_5 \rtimes_\varphi \mathbb{Z}_4$

The generators of the four-dimensional representation and the character table are given in Tab. 1. The Clebsch-Gordan coefficients for the Kronecker product 4×4 can be calculated following the algorithm described in [68]. The Kronecker product of $a = (a_1,\, a_2,\, a_3,\, a_4) \sim \underline{\mathbf{4}}$ and $b = (b_1,\, b_2,\, b_3,\, b_4) \sim \underline{\mathbf{4}}$ results in

$$
\begin{aligned}
\underline{\mathbf{1}}_{\mathbf{1}} &\sim \frac{1}{2}\left(a_1 b_3 + a_3 b_1 + a_2 b_4 + a_4 b_2\right)\,, &\qquad \underline{\mathbf{1}}_{\mathbf{2}} &\sim \frac{1}{2}\left(a_1 b_3 + a_3 b_1 - a_2 b_4 - a_4 b_2\right)\,,\\
\underline{\mathbf{1}}_{\mathbf{3}} &\sim \frac{1}{2}\left(a_1 b_3 - a_3 b_1 + \mathrm{i}\, a_2 b_4 - \mathrm{i}\, a_4 b_2\right)\,, &\qquad \underline{\mathbf{1}}_{\mathbf{4}} &\sim \frac{1}{2}\left(a_1 b_3 - a_3 b_1 - \mathrm{i}\, a_2 b_4 + \mathrm{i}\, a_4 b_2\right)\,,
\end{aligned}
\tag{69}
$$

$$
\begin{aligned}
\underline{\mathbf{4}}_S &\sim \frac{1}{\sqrt{2}}\left(a_3 b_4 + a_4 b_3,\quad a_4 b_1 + a_1 b_4,\quad a_1 b_2 + a_2 b_1,\quad a_2 b_3 + a_3 b_2\right)\,,\\
\underline{\mathbf{4}}_S &\sim \left(a_2 b_2,\quad a_3 b_3,\quad a_4 b_4,\quad a_1 b_1\right)\,,\\
\underline{\mathbf{4}}_A &\sim \frac{1}{\sqrt{2}}\left(a_3 b_4 - a_4 b_3,\quad a_4 b_1 - a_1 b_4,\quad a_1 b_2 - a_2 b_1,\quad a_2 b_3 - a_3 b_2\right)\,.
\end{aligned}
\tag{70}
$$

The Clebsch-Gordan coefficients for $\underline{\mathbf{1}}_{\mathbf{i}} \times \underline{\mathbf{4}}$ with $\alpha \sim \underline{\mathbf{1}}_{\mathbf{i}}$ and $(b_1,\, b_2,\, b_3,\, b_4) \sim \underline{\mathbf{4}}$ are

$$
\begin{aligned}
\underline{\mathbf{1}}_{\mathbf{1}} \times \underline{\mathbf{4}} &\simeq \left(\alpha\, b_1,\quad \alpha\, b_2,\quad \alpha\, b_3,\quad \alpha\, b_4\right)\,, & \underline{\mathbf{1}}_{\mathbf{2}} \times \underline{\mathbf{4}} &\simeq \left(\alpha\, b_1,\quad -\alpha\, b_2,\quad \alpha\, b_3,\quad -\alpha\, b_4\right)\,,\\
\underline{\mathbf{1}}_{\mathbf{3}} \times \underline{\mathbf{4}} &\simeq \left(\alpha\, b_1,\quad -\mathrm{i}\,\alpha\, b_2,\quad -\alpha\, b_3,\quad \mathrm{i}\,\alpha\, b_4\right)\,, & \underline{\mathbf{1}}_{\mathbf{4}} \times \underline{\mathbf{4}} &\simeq \left(\alpha\, b_1,\quad \mathrm{i}\,\alpha\, b_2,\quad -\alpha\, b_3,\quad -\mathrm{i}\,\alpha\, b_4\right)\,,
\end{aligned}
\tag{71}
$$

(a) Character table

	classes				
	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$
G	$\mathbb{1}$	s	s^2	t	s^3
$h_{\mathcal{C}_i}$	1	4	2	5	4
$\underline{\mathbf{1}}_{\mathbf{1}}$	1	1	1	1	1
$\underline{\mathbf{1}}_{\mathbf{2}}$	1	-1	1	1	-1
$\underline{\mathbf{1}}_{\mathbf{3}}$	1	$-\mathrm{i}$	-1	1	i
$\underline{\mathbf{1}}_{\mathbf{4}}$	1	i	-1	1	$-\mathrm{i}$
$\underline{\mathbf{4}}$	4	0	0	-1	0

(b) Breaking patterns

subgroup	VEV configuration
D_5	$\langle \underline{\mathbf{1}}_{\mathbf{2}} \rangle$
$\mathbb{Z}_5$	$\langle \underline{\mathbf{1}}_{\mathbf{3}} \rangle$
$\mathbb{Z}_5$	$\langle \underline{\mathbf{1}}_{\mathbf{4}} \rangle$
$\mathbb{Z}_4$	$\langle \underline{\mathbf{4}} \rangle \sim \big(1,\ 1,\ 1,\ 1\big)$
$\mathbb{Z}_4$	$\langle \underline{\mathbf{4}} \rangle \sim \big(\eta^4,\ \eta,\ \eta^2,\ 1\big)$
$\mathbb{Z}_4$	$\langle \underline{\mathbf{4}} \rangle \sim \big(\eta^3,\ \eta^2,\ \eta^4,\ 1\big)$
$\mathbb{Z}_4$	$\langle \underline{\mathbf{4}} \rangle \sim \big(\eta^2,\ \eta^3,\ \eta,\ 1\big)$
$\mathbb{Z}_4$	$\langle \underline{\mathbf{4}} \rangle \sim \big(\eta,\ \eta^4,\ \eta^3,\ 1\big)$
$\mathbb{Z}_2$	$\langle \underline{\mathbf{4}} \rangle \sim \big(1,\ 0,\ 1,\ 0\big), \big(0,\ 1,\ 0,\ 1\big)$
$\mathbb{Z}_2$	$\langle \underline{\mathbf{4}} \rangle \sim \big(\eta^2,\ 0,\ 1,\ 0\big), \big(0,\ \eta,\ 0,\ 1\big)$
$\mathbb{Z}_2$	$\langle \underline{\mathbf{4}} \rangle \sim \big(\eta^4,\ 0,\ 1,\ 0\big), \big(0,\ \eta^2,\ 0,\ 1\big)$
$\mathbb{Z}_2$	$\langle \underline{\mathbf{4}} \rangle \sim \big(\eta,\ 0,\ 1,\ 0\big), \big(0,\ \eta^3,\ 0,\ 1\big)$
$\mathbb{Z}_2$	$\langle \underline{\mathbf{4}} \rangle \sim \big(\eta^3,\ 0,\ 1,\ 0\big), \big(0,\ \eta^4,\ 0,\ 1\big)$

(c) Generators of $\underline{\mathbf{4}}$

$$s = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad t = \begin{pmatrix} \eta & 0 & 0 & 0 \\ 0 & \eta^4 & 0 & 0 \\ 0 & 0 & \eta^2 & 0 \\ 0 & 0 & 0 & \eta^3 \end{pmatrix}$$

Table 1: Group theoretical details of SG$(20,3)$: Character table in Tab. (a), breaking patterns in Tab. (b), and the generators of $\underline{\mathbf{4}}$ in Tab. (c). G denotes the generating element, $h_{\mathcal{C}_i}$ is the order of the elements and $\eta = \exp(2\pi\mathrm{i}/5)$ is the fifth root of unity.

where S and A indicate that the representation is in the symmetric or antisymmetric part, respectively. The different possible breaking patterns of $\mathrm{SG}(20,3)$ to its subgroups are shown in Tab. 1, where we have used the algorithm described in Appendix C of [69].

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